



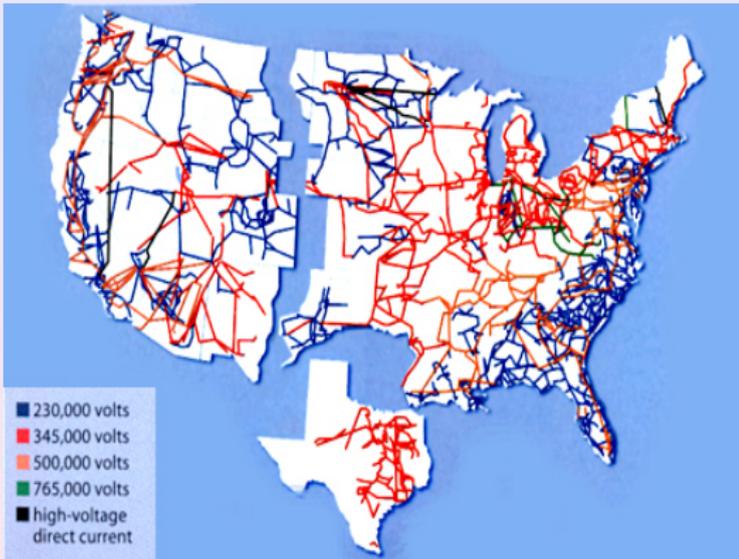
Models and Control of Collective Spatio-Temporal Phenomena in Power Grids

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and New Mexico Consortium

SIAM DS 2011, Snowbird, May 24, 2011

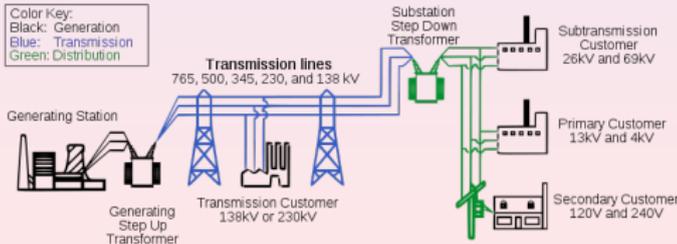
Supported by LDRD/LANL/DOE, DTRA and NSF



US power grid

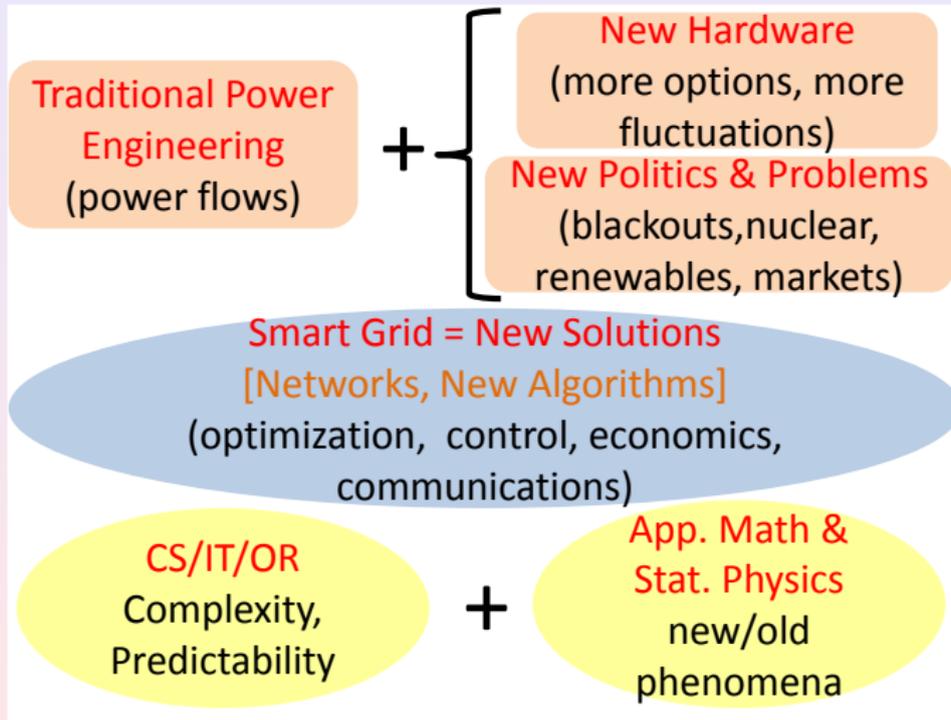
The greatest
Engineering
Achievement of
the 20th century

Color Key:
Black: Generation
Blue: Transmission
Green: Distribution



will require
smart revolution
in the 21st century

Smart Grid



Preliminary Remarks

The power grid operates according to **AC electrodynamics**

- Transmission vs Distribution. Generators vs Loads.
- Dynamics is associated with electro-mechanical effects, customers and control
- **Many Scales**
- Loads Fluctuates. Graph changes. Renewables, Electric Vehicles new realities \Rightarrow even more fluctuations

Many Scales Involved

Power & Voltage

- **1KW** - typical household; **$10^3 \text{KW} = 1\text{MW}$** - consumption of a medium-to-large residential, commercial building; **$10^6 \text{KW} = 1\text{GW}$** -large unit of a Nuclear Power plant (30GW is the installed wind capacity of Germany =8% of total, US wind penetration is 5%- [30% by 2030?]); **$10^9 \text{KW} = 1\text{TW}$** - US capacity
- Distribution - **4 – 13KV**. Transmission - **100 – 1000KV**.

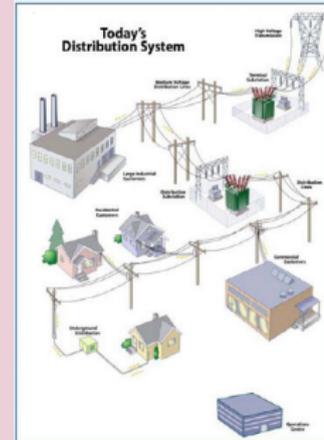
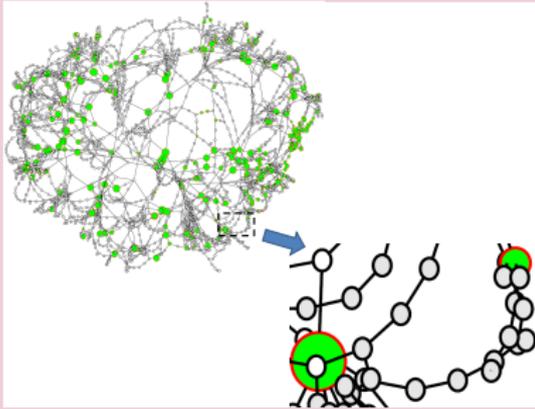
Temporal Scales [control is getting faster]

- **17ms** -AC (60Hz) period, target for Phasor Measurement Units sampling rate (10-30 measurements per second)
- **1s** - electro-mechanical wave [motors induced] propagates $\sim 500\text{km}$
- **2-10s** - SCADA delivers measurements to control units
- **$\sim 1 \text{ min}$** - loads change (demand response), wind ramps, etc (**toughest scale to control**)
- **5-15min** - state estimations are made (for markets), voltage collapse
- **up to hours** - maturing of a cascading outage over transmission grids

My tasks for today

- Give Applied Math/Physics background/intuition on Power Flows and related phenomena, e.g. **voltage collapse**
- Discuss new problems and challenges in Smart Grids
- ... related to **control**
- ... **extreme fluctuations** and resulting contingencies

Linear Segments in Transmission & Distribution



- Spatially Continuous (ODE) Model of a Linear Segment
- Dynamics & Control of Loads
- Critical Slow Down & Voltage Collapse
- Structural and Dynamic (PDE) Stability

Applied Math/Physics Perspective (to appear soon)

- MC, S. Backhaus, K. Turitsyn, V. Chernyak, V. Lebedev

Basic AC Power Flow Equations (Static)

The Kirchhoff Laws (linear)

$$\forall a: \sum_{b \sim a} J_{ab} = J_a \text{ for currents}$$

$$\forall(a, b): J_{ab} z_{ab} = V_a - V_b \text{ for potentials}$$

Complex Power Flows [balance of power, nonlinear, static]

$$\forall a: p_a + iq_a = V_a J_a^* = V_a \sum_{b \sim a} J_{ab}^* = V_a \sum_{b \sim a} \frac{V_a^* - V_b^*}{z_{ab}^*}$$

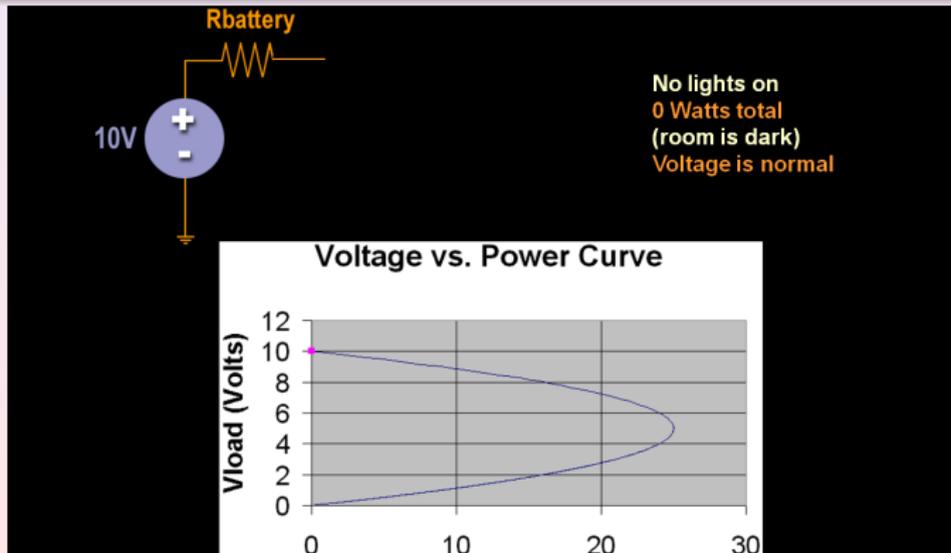
- Nonlinear in terms of Real and Reactive powers
- Known parameters: different (injection/consumption/control) conditions on generators (p, v) and loads (p, q)
- The task is to find the unknown (flows and potentials)

$$V = v \exp(i\theta), \quad \underbrace{z}_{\text{impedance}} = \underbrace{r}_{\text{resistance}} + i \underbrace{x}_{\text{inductance}}, \quad \underbrace{z^{-1}}_{\text{admittance}} = \underbrace{g}_{\text{conductance}} + i \underbrace{\beta}_{\text{susceptance}}$$

Voltage Collapse

- **Voltage Collapse** = Power Flow Eqs. have no solution(s)

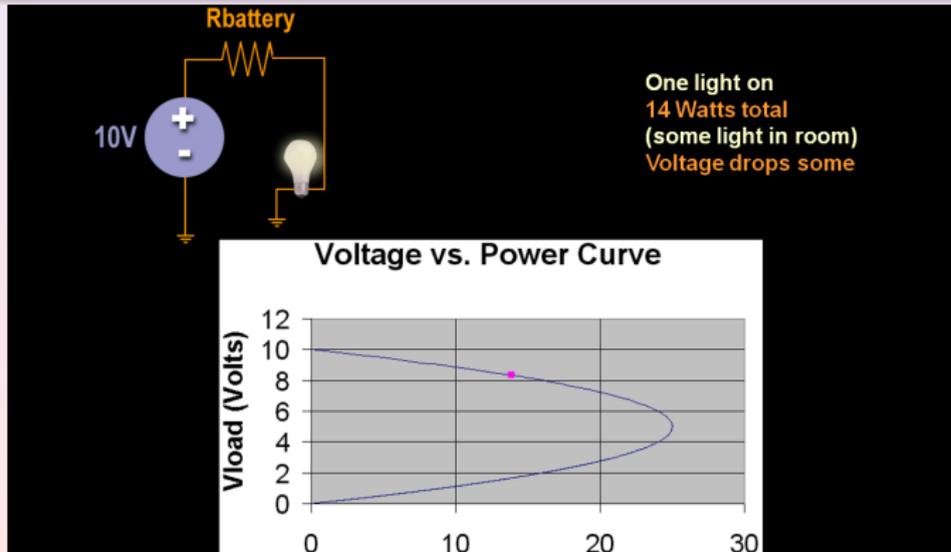
Animation of Voltage Collapse (by P.W. Sauer)



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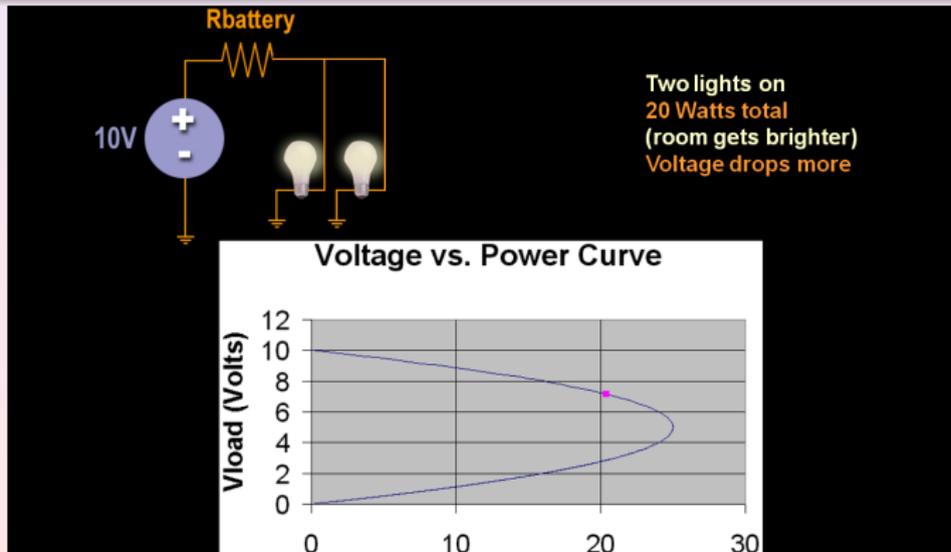
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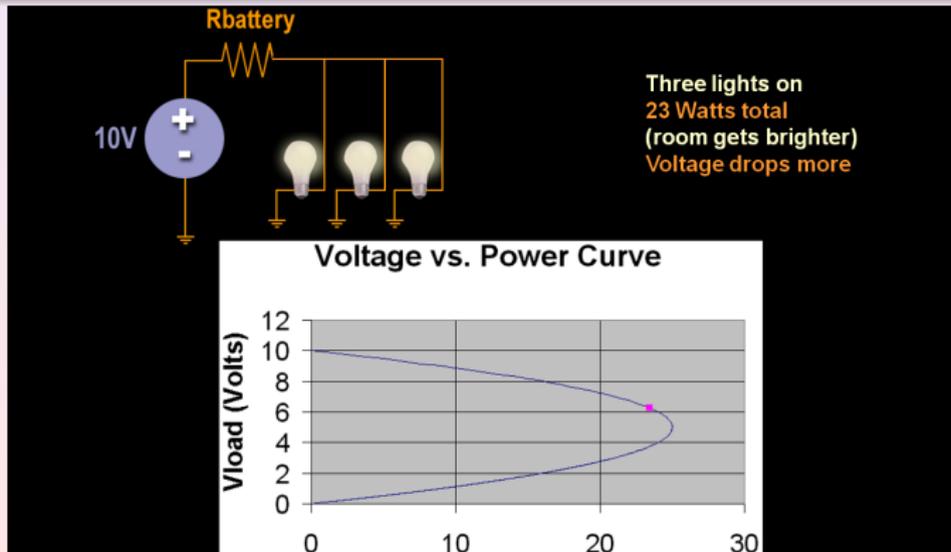
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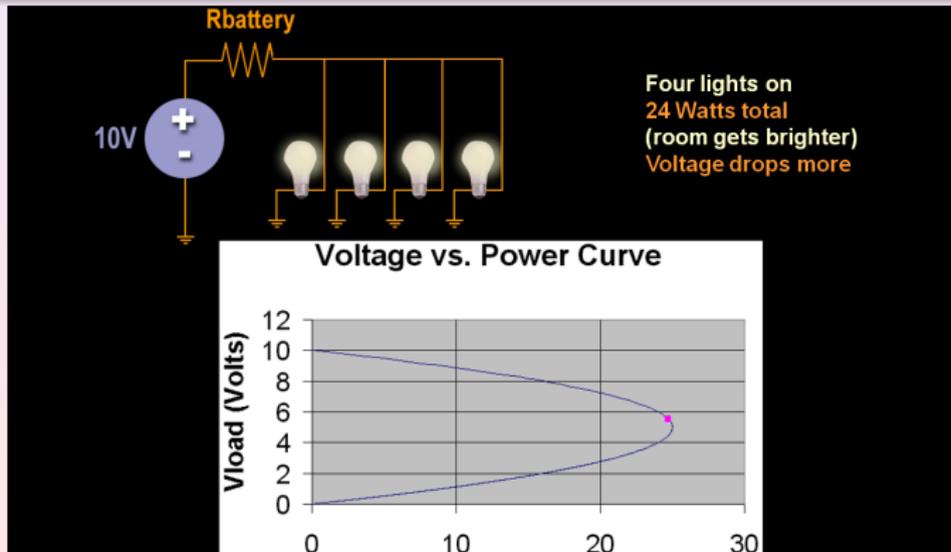
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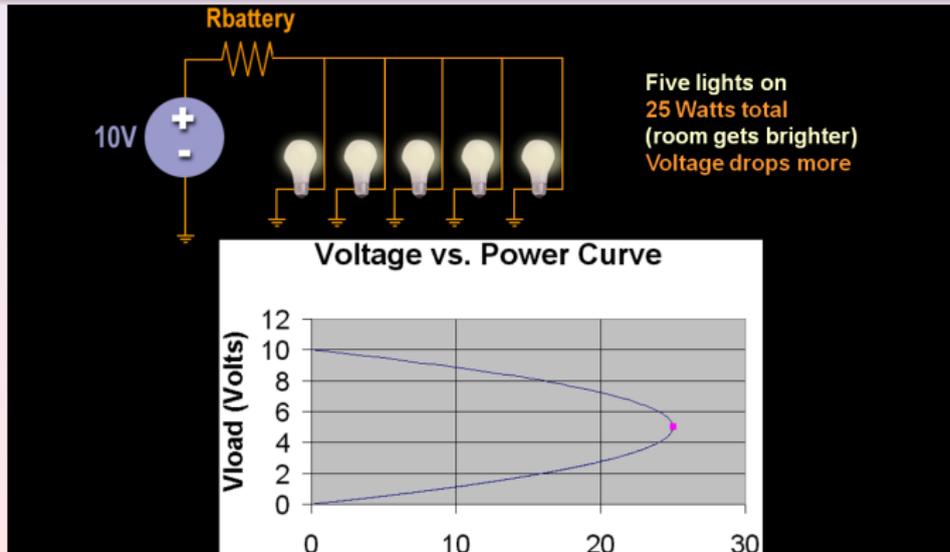
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Voltage Collapse

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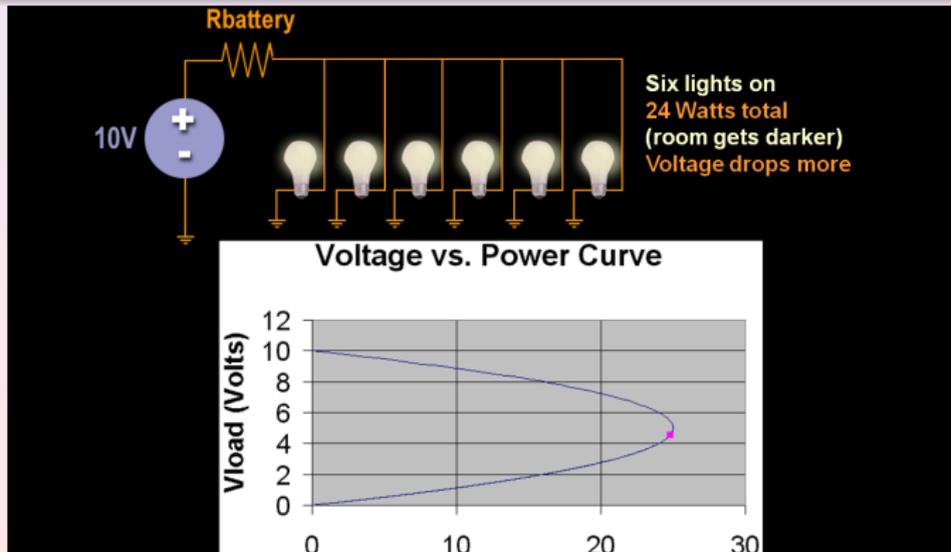
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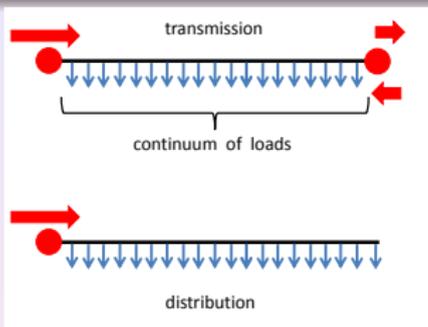
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Continuum (one dimensional) power flows



Boundary Conditions: $v(0) = 1$, $\theta(0) = 0$ +

- $P(0)$ and $v(L)$ are fixed
- $P(L) = Q(L) = 0$

From Algebraic Eqs. on a (linear) Graph to Power Flow ODEs

$$0 = \underbrace{p + \beta \partial_r (v^2 \partial_r \theta) + gv (\partial_r^2 v - v (\partial_r \theta)^2)}_{\text{balance of real power}}, \quad 0 = \underbrace{q + \beta v (\partial_r^2 v - v (\partial_r \theta)^2) - g \partial_r (v^2 \partial_r \theta)}_{\text{balance of reactive power}}$$

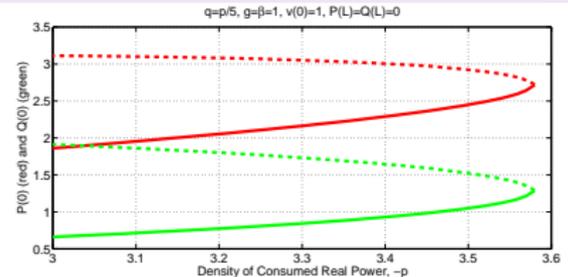
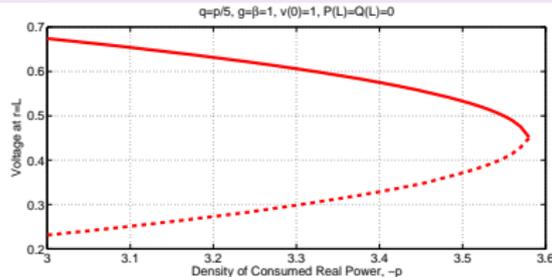
$$P = \underbrace{-\beta v^2 \partial_r \theta - gv \partial_r v}_{\text{real power flowing through the segment}}, \quad Q = \underbrace{-\beta v \partial_r v + g v^2 \partial_r \theta}_{\text{reactive power flowing through the segment}}$$

$$0 = \underbrace{p}_{\text{real consumption}} - \underbrace{\partial_r P}_{\text{real transport}} - \underbrace{r \frac{P^2 + Q^2}{v^2}}_{\text{real dissipation}}, \quad 0 = \underbrace{q}_{\text{reactive consumption}} - \underbrace{\partial_r Q}_{\text{reactive transport}} - \underbrace{x \frac{P^2 + Q^2}{v^2}}_{\text{reactive dissipation}}$$

Distribution Feeder: Nose Curve

Boundary Value Analysis for a Feeder

$$v(0) = 1, \theta(0) = 0, p, q \text{ const}; L \text{ is fixed}; \partial_r P(L) = \partial_r Q(L) = 0$$



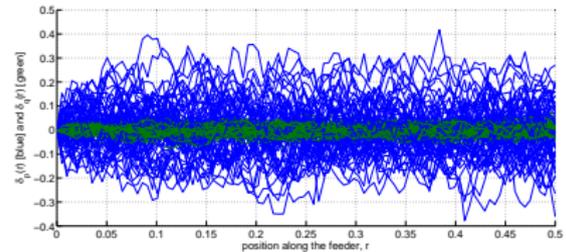
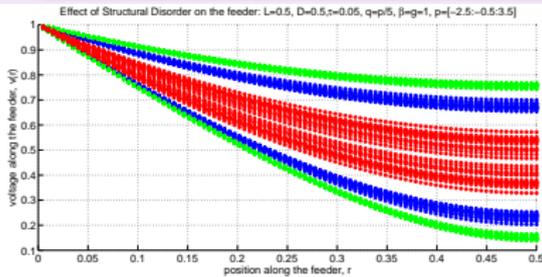
- “Nose curve” in the standard (in power engineering) v - p plane
- Power which needs to be injected is smaller for stable solution (a variational principle of a kind)

► Linear Segment in Transmission

Effects of Structural Disorder

Amplification and Spread of Disorder

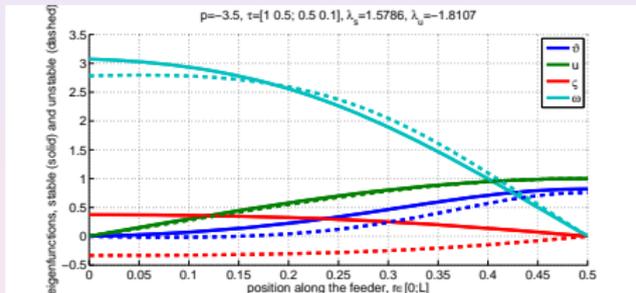
The same feeder ... with quenched disorder in p, q



- In spite of the the fact that the amount of disorder in p and q added was identical in all the three cases, the spread in voltage was significantly stronger close to criticality (the point of voltage collapse).
- The disorder is smoothed out (distributed) in voltage profiles.

Dynamic (small signal) Stability

Boundary Value Spectral Analysis (Linearized PDEs)



- Phenomenological (falsifiable) Model for Dynamics of Loads (p, q) dependence on (response to) $(\theta, v) +$ Load Control (based on local measurements)
- $0 \rightarrow \tau_{\theta\theta}\partial_t\theta + \tau_{\theta v}\partial_tv = p - p_{el}$
 $0 \rightarrow \tau_{v\theta}\partial_t\theta + \tau_{vv}\partial_tv = q - q_{el}$
- $\hat{\tau}, p, q$ (and their v dependence) should be “learned” from measurements

Critical slow-down and Long-range correlations

- $\sim \exp(-t\lambda) * \Psi_\lambda(r)$
- $\lambda_{\text{stable}} > 0, \lambda_{\text{unstable}} < 0; \lambda \rightarrow 0$ at the criticality
- $\Psi_\lambda(r)$ is correlated on the feeder size, L

Conclusions & Path Forward (Voltage Collapse)

- ODE-PDE approach is useful tool of model reduction (coarse-graining)
- Approaching voltage collapse is similar to spinodal/bifurcation point (allows interpretation in terms of “energy landscape”)
- Slowdown precedes voltage collapse (and, possibly, cascades)
- Disorder is amplified close to collapse

The ODE-PDE formalism allows to account for ...

- Nonlinear regime(s) of the collapse (more realistic modeling of load dynamics and control)
- Stochastic (temporal) effects ... driven non-equilibrium system
- Two dimensional modeling (multiple generators with inertia, e.g. of Eastern Interconnect)
- Electro-mechanical waves, inertia, dispersion, non-linearity (extending ODE approach of Thorp et al '98) ... “power grid spectroscopy” based on measurements & visualization (joint project with T. Overbye)
- Synchronization phenomena (Dörfler & Bullo '10-'11)
- Inverse cascade of phase fluctuations (Mezic et al '10-'11)

Outline

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 - Intermittent Failures: Examples

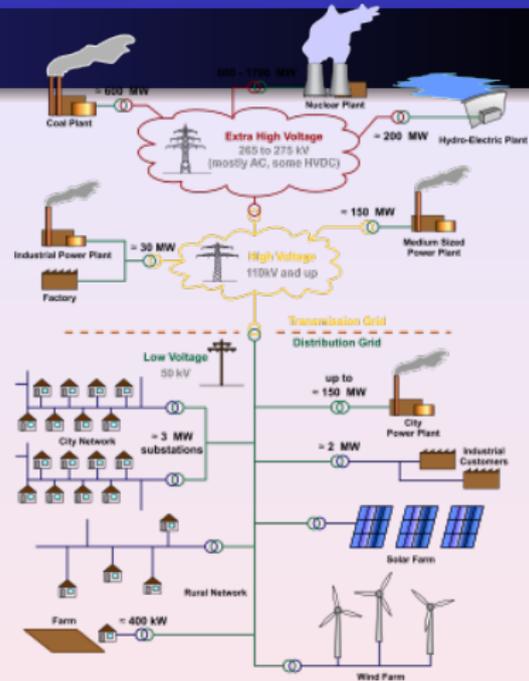
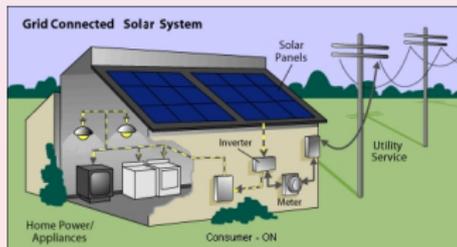
K. Turitsyn (MIT), P. Sulc (Oxford), S. Backhaus and MC (LANL)

- *Optimization of Reactive Power by Distributed Photovoltaic Generators*, to appear in Proceedings of the IEEE, special issue on Smart Grid (2011), <http://arxiv.org/abs/1008.0878>
- *Local Control of Reactive Power by Distributed Photovoltaic Generators*, proceedings of IEEE SmartGridComm 2010, <http://arxiv.org/abs/1006.0160>
- *Distributed control of reactive power flow in a radial distribution circuit with high photovoltaic penetration*, IEEE PES General Meeting 2010 (invited to a super-session), <http://arxiv.org/abs/0912.3281>



Setting & Question & Idea

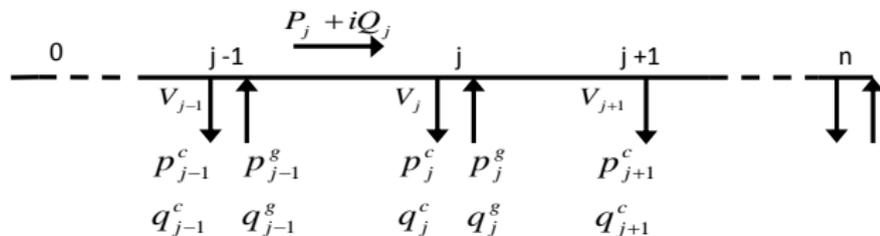
- Distribution Grid (old rules, e.g. voltage is controlled only at the point of entrance)
- Significant Penetration of Photovoltaic (new reality)
- How to control swinging/fluctuating voltage (reactive power)?



Idea(s)

- Use Inverters.
- Control Locally.

Losses vs Voltage



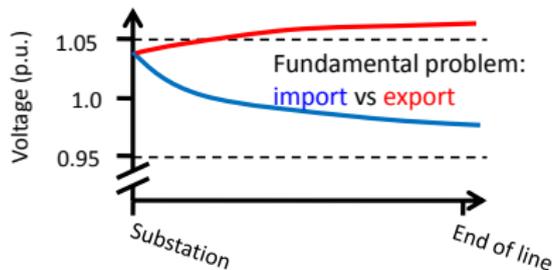
$$Loss_j = r_j \frac{P_j^2 + Q_j^2}{V_0^2}$$

$$\Delta V_j = -(r_j P_j + x_j Q_j)$$

Competing objectives

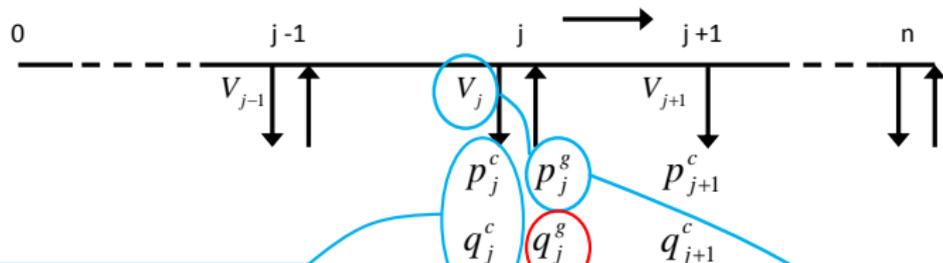
Minimize losses $\rightarrow Q_j=0$

Voltage regulation $\rightarrow Q_j = -(r_j/x_j)P_j$



- Rapid reversal of real power flow can cause undesirably large voltage changes
- Rapid PV variability cannot be handled by current electro-mechanical systems
- Use PV inverters to generate or absorb reactive power to restore voltage regulation
- In addition... optimize power flows for minimum dissipation

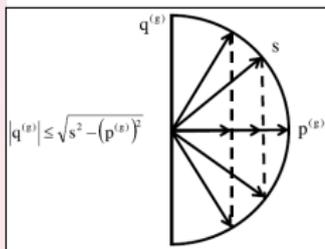
Parameters available and Limits for Control



Not available to affect control
 —but available
 (via advanced metering)
 for control input

Available—minimal impact on
 customer, extra inverter duty

Not available to affect control — but
 available (via inverter Point of Common
 Coupling) for control input



Schemes of Control

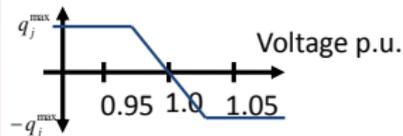
- Base line (do nothing)

$$q_j^g = 0$$

- Unity power factor

$$q_j^g = q_j^c \quad F^{(L)}$$

- Proportional Control
(EPRI white paper)



- voltage control heuristics

$$q_j^g = q_j^c + \frac{r_j}{x_j} (p_j^c - p_j^g)$$

- composite control

$$\begin{aligned} q_j^g &= Kq_j^c + (1-K)[q_j^c + \frac{r_j}{x_j} (p_j^c - p_j^g)] \\ &= KF_j^{(L)} + (1-K)F_j^{(V)} \end{aligned}$$

- Hybrid (composite at V=1 built in proportional)

$$q_j^g = F_j(K) + (q_j^{\max} - F_j(K)) \left(1 - \frac{2}{1 + \exp(-4(V_j - 1)/\delta)} \right)$$

$$F_j(K) = \text{Constr}_j(KF_j^{(L)} + (1-K)F_j^{(V)})$$

$$\text{Constr}_j[q] = \begin{cases} q, & |q| \leq q_j^{\max} \\ (q/|q|)q_j^{\max}, & \text{otherwise} \end{cases}$$

Prototypical Distribution Circuit: Case Study

Import—Heavy cloud cover

- p^c = uniformly distributed 0-2.5 kW
- q^c = uniformly distributed $0.2p^c$ - $0.3p^c$
- $p^g = 0$ kW
- Average import per node = 1.25 kW

Export—Full sun

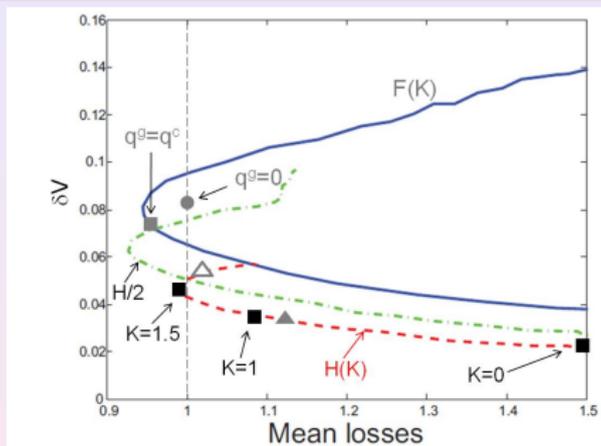
- p^c = uniformly distributed 0-1.0 kW
- q^c = uniformly distributed $0.2p^c$ - $0.3p^c$
- $p^g = 2.0$ kW
- Average export per node = 0.5 kW

- $V_0=7.2$ kV line-to-neutral
- $n=250$ nodes
- Distance between nodes = 200 meters
- Line impedance = $0.33 + i 0.38 \Omega/\text{km}$
- 50% of nodes are PV-enabled with 2 kW maximum generation
- Inverter capacity $s=2.2$ kVA – 10% excess capacity

Measures of control performance

- δV —maximum voltage deviation in transition from export to import
- Average of import and export circuit dissipation relative to “Do Nothing-Base Case”

Reactive Control of a Feeder: Conclusions

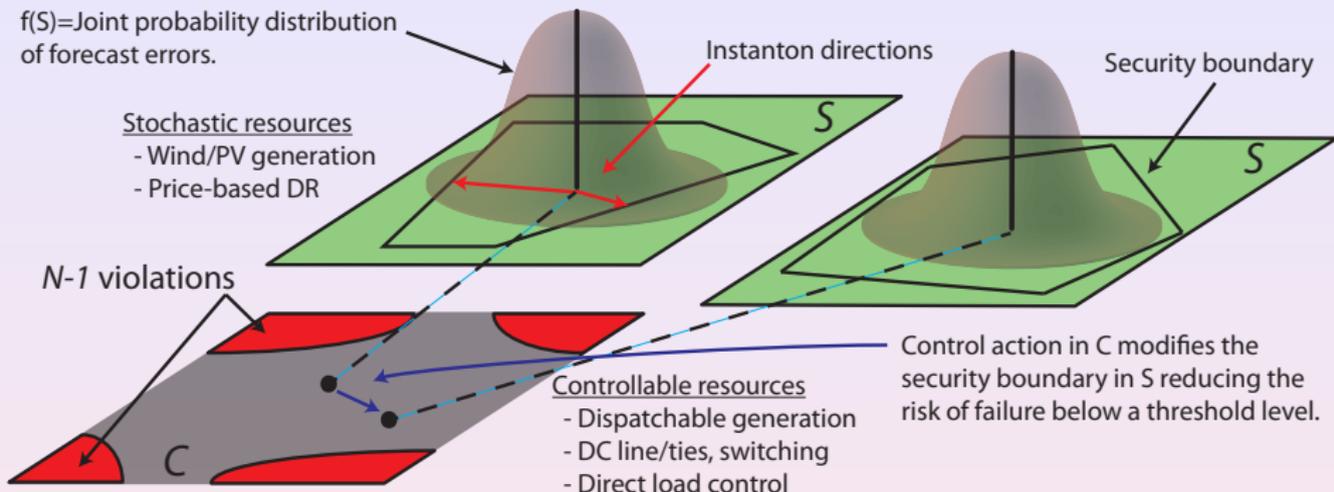


- Composite Control
- Hybrid "1/2" Control
- Hybrid "1" Control

- Equitable division of reactive generation duty and adequate voltage regulation will be difficult to ensure simultaneously.
- All local inputs p_c , q_c , p_g and v should be considered for control of q_g . Hybrid/blended control shows improved performance and allows for simple tuning of the control to different conditions.
- Adequate voltage regulation and reduction in circuit dissipation can be achieved by local inverter-based control of reactive generation

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- New probabilistic paradigm for identification and control of security boundary [scheme above is from LANL ARPA-E/DOE proposal led by S. Backhaus]
- Focus of this discussion: **finding instanton - probabilistic most dangerous instance - efficiently**

MC, F. Pan (LANL) and M. Stepanov (UA Tucson)

- Predicting Failures in Power Grids: The Case of Static Overloads, IEEE Transactions on Smart Grids 2, 150 (2010).



MC, FP, MS & R. Baldick (UT Austin)

- Exact and Efficient Algorithm to Discover Extreme Stochastic Events in Wind Generation over Transmission Power Grids, invited session on Smart Grid Integration of Renewable Energy at CDC/ECC 2011.

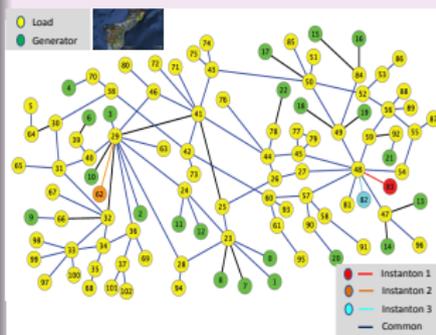


Failure Probability

- Normally the grid is ok (SATisfied) ... but sometimes failures (UNSATisfied) happens
- How to estimate failure probability (UNSAT)?

Static overload

- Power Flows. Control=Generation Dispatch.
Constraints = Thermal and Generation
- Probabilistic Forecast of Loads (given)
- **SAT**= Load shedding is avoidable;
UNSAT=load shedding is unavoidable
- Find the most probable **UNSAT** configuration of loads



Extreme Statistics of Failures

- Statistics of loads/demands is assumed given: $\mathcal{P}(\mathbf{d})$
- $\mathbf{d} \in \text{SAT} = \text{No Shedding}$; $\mathbf{d} \in \text{UNSAT} = \text{Shedding}$

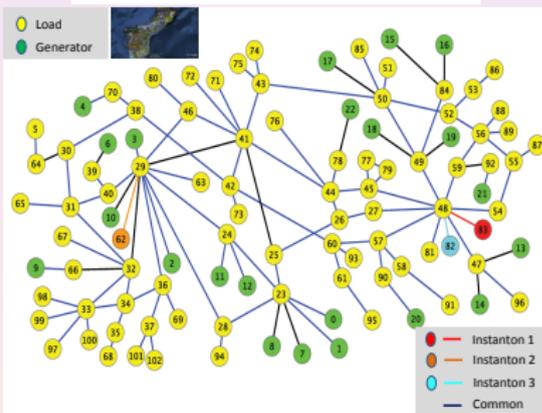
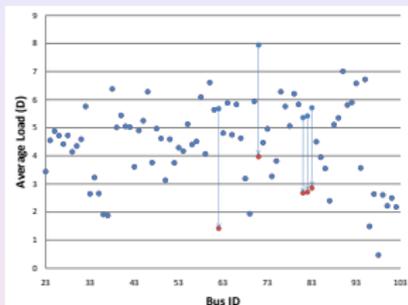
Most Dangerous Configuration of the demand = the Instanton

- $\arg \max_{\mathbf{d}} \mathcal{P}(\mathbf{d})|_{\mathbf{d} \notin \text{SAT}}$ - most probable instanton
- SAT is a polytope (finding min-shedding solution is an );
– $-\log(\mathcal{P}(\mathbf{d}))$ is (typically) convex

The task: to find the (rated) list of (local) instantons

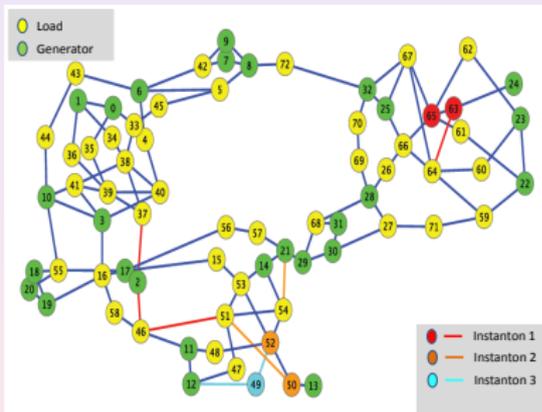
- The most probable instanton represents the large deviation asymptotic of the failure probability
- Use an efficient heuristics to find candidate instantons (technique was borrowed from our previous “rare events” studies of a similar problem in error-correction '04-'11)

Example of Guam



- Gaussian Statistics of demands (input) leads to **Intermittency** (output) = instantons (rare, UNSAT) are distinctly different from normal (typical, SAT)
- **The instantons are sparse** (difference with “typical” is localized on troubled nodes)
- The troubled nodes are repetitive in multiple-instantons
- Violated constraints (edges) are next to the troubled nodes
- Instanton structure is not sensitive to small changes in statistics of demands

Example of IEEE RTS96 system



- The instantons are well localized (but still not sparse)
- The troubled nodes and structures are repetitive in multiple-instantons
- Violated constraints (edges) can be far from the troubled nodes: **long correlations**
- Instanton structure is not sensitive to small changes in statistics of demands

▶ Wind Contingency

Path Forward (for predicting failures)

Path Forward

- Many large-scale practical tests, e.g. ERCOT wind integration
- The instanton-amoeba allows upgrade to other (than LP_{DC}) network stability testers, e.g. for AC flows and transients
- Instanton-search can be accelerated, utilizing LP-structure of the tester (exact & efficient for low-dimensional control). The exactness can probably be extended beyond LP-DC.
- New paradigm for **instanton-based** identification and control of security boundary

Bottom Line

- A lot of interesting **collective phenomena** in the power grid settings for Applied Math, Physics, CS/IT analysis
- The research is timely (blackouts, renewables, stimulus)

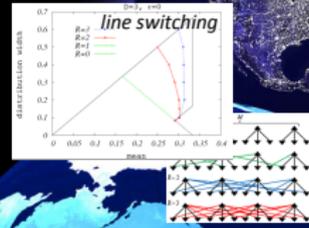
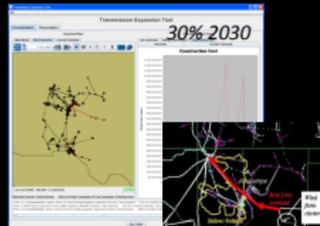
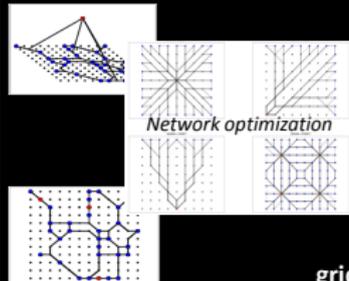
Other Problems we are working on

- Efficient PHEV charging via queuing/scheduling with and without communications and delays
- Power Grid Spectroscopy (power grid as a medium, electro-mechanical waves and their control, voltage collapse, dynamical state estimations)
- Effects of Renewables (intermittency of winds, clouds) on the grid & control
- Load Control, scheduling with time horizon (dynamic programming +)
- Price Dynamics & Control for the Distribution Power Grid
- Post-emergency Control (restoration and de-islanding)

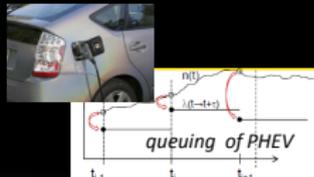
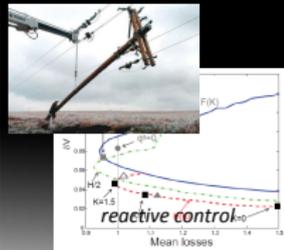
For more info - check:

<http://cnls.lanl.gov/~chertkov/SmarterGrids/>
<https://sites.google.com/site/mchertkov/projects/smart-grid>

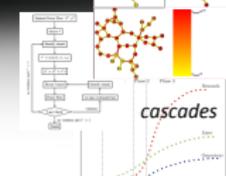
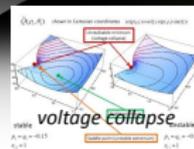
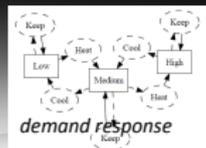
LANL LDRD DR (FY09-11): Optimization & Control Theory for Smart Grids



grid planning



grid control



<http://cnls.lanl.gov/~chertkov/SmarterGrids/>

Dynamical Systems Approaches in Smart Grids I and II

Part I, MS78, 3:00pm-4:40pm, Ballroom I

- 3:00-3:20 Critical Slowing Down As An Indicator of Dynamic Instability in Power Systems, **P. Hines** and E. Cotilla-Sanchez
- 3:25-3:45 Inverse Problems in Power System Dynamics, **I. Hiskens**
- 3:50-4:10 Cascading Dynamics of Power Grid Networks, **K. Turitsyn**
- 4:15-4:35 Algebraic Methods for Robust Power Grid Analysis and Design, **M. Anghel**

Part II, MS89, 5:10pm-6:50pm, Ballroom I

- 5:10-5:30 Modeling and Control of Aggregated Heterogeneous Thermostatically Controlled Loads for Ancillary Services, **D. Callaway**, S. Koch, J. Mathieu
- 5:35-5:55 [canceled] Modeling and Simulation of a Renewable and Resilient Electric Power Grid, **T. Overbye**
- 6:00-6:20 Rules Versus Optimization for Enabling Adaptive Network Topologies, **S. Blumsack**
- 6:25-6:45 Demand Response to Uncertainty in Renewable Energy, S. Low and **L. Jiang**



Thank You!

Energy Functional Landscape. Voltage Collapse.

Transmission ($r \ll x$): PF solutions are minima of the Functional

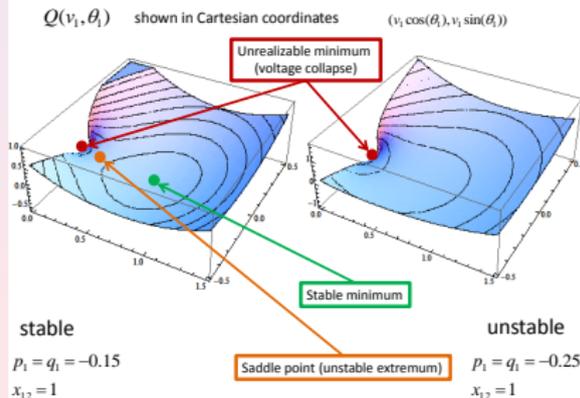
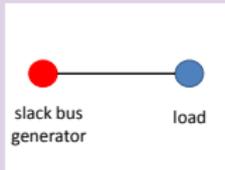
reactive power "lost" in lines

$$Q(\mathbf{v}, \boldsymbol{\theta}) = \sum_{\{a,b\} \in \mathcal{G}_1} \frac{v_a^2 + v_b^2 - 2v_a v_b \cos(\theta_a - \theta_b)}{2x_{ab}} - \sum_{a \in \mathcal{G}_0} \theta_a p_a - \sum_{a \in \mathcal{G}_{\text{loads}}} \log(v_a) q_a$$

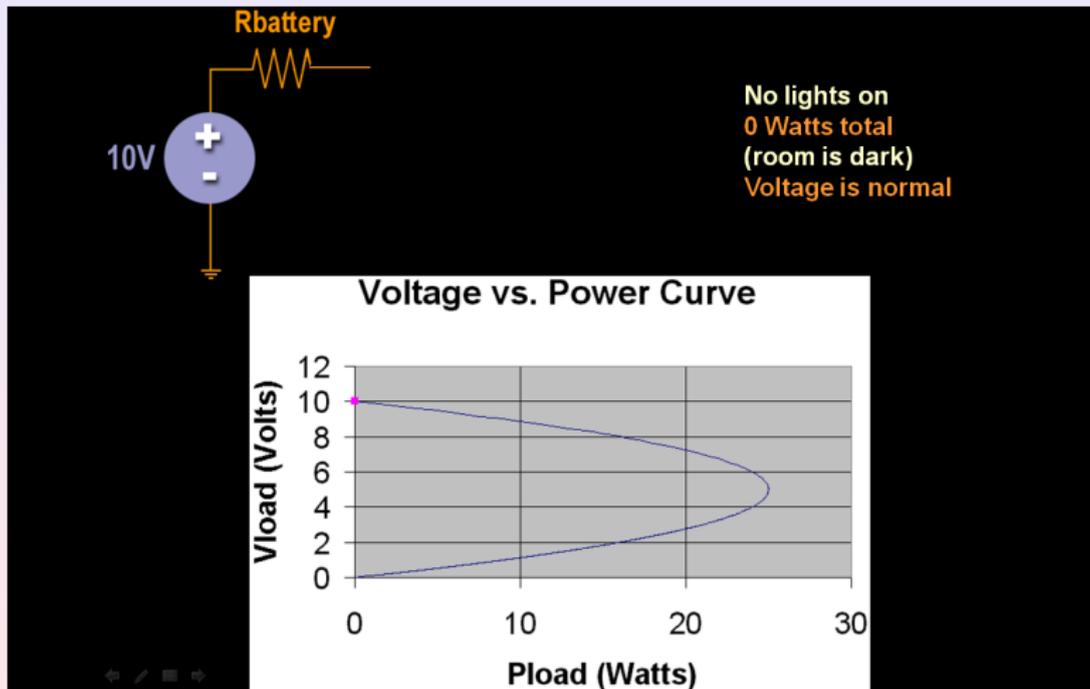
- ▶ Voltage Collapse= PF eqs have no solution(s); $Q(\mathbf{v}, \boldsymbol{\theta})$ has no extrema

Example: Single Load (p_1, q_1)
and Slack Bus ($v_0 = 1, \theta_0 = 0$)

$$Q = \frac{1 + v_1^2 - 2v_1 \cos(\theta_1)}{2x} - \theta_1 p_1 - \log(v_1) q_1$$



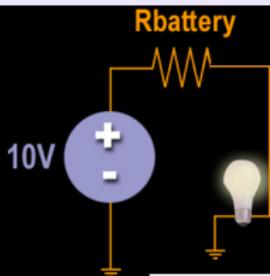
Voltage Collapse Animation (P.W. Sauer)



◀ Power Flows

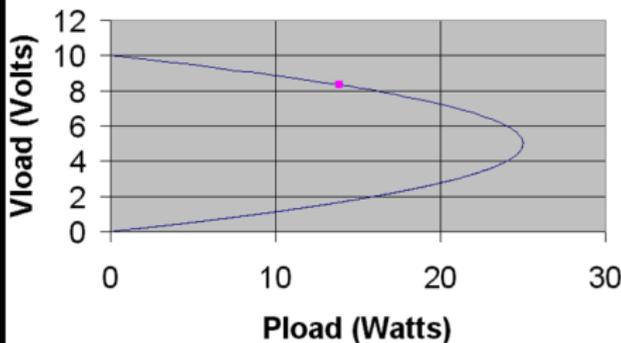


Voltage Collapse Animation (P.W. Sauer)



One light on
14 Watts total
(some light in room)
Voltage drops some

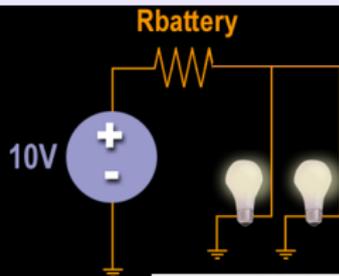
Voltage vs. Power Curve



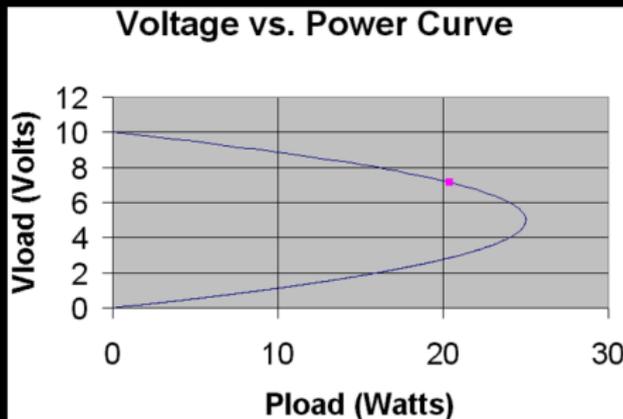
◀ Power Flows



Voltage Collapse Animation (P.W. Sauer)



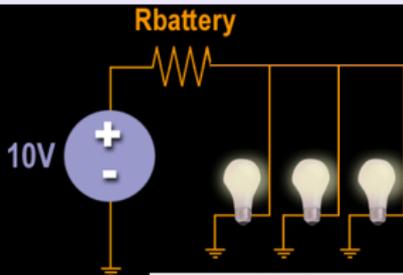
Two lights on
20 Watts total
(room gets brighter)
Voltage drops more



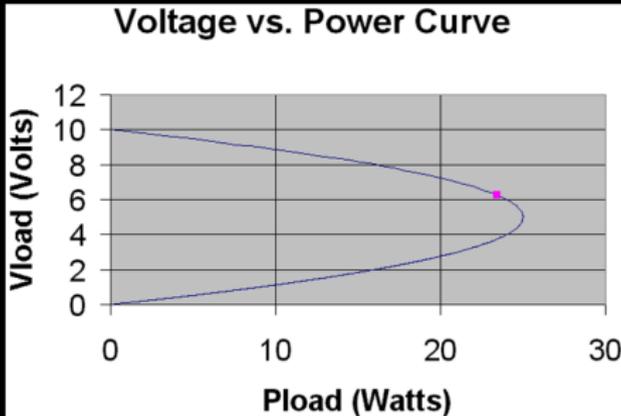
◀ Power Flows



Voltage Collapse Animation (P.W. Sauer)



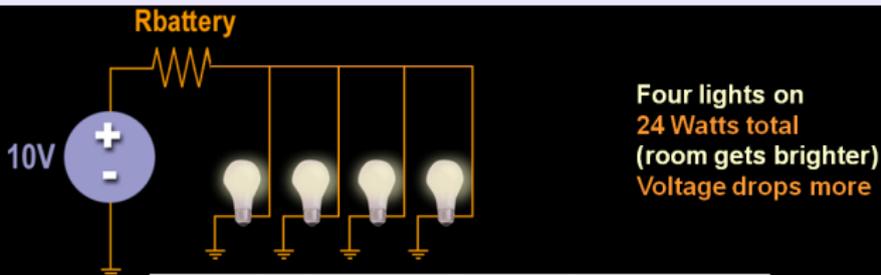
Three lights on
23 Watts total
(room gets brighter)
Voltage drops more



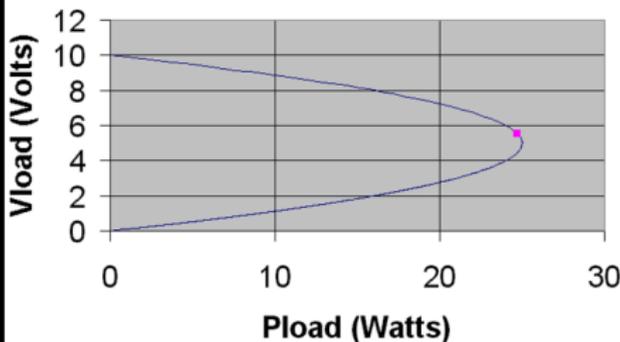
◀ Power Flows



Voltage Collapse Animation (P.W. Sauer)



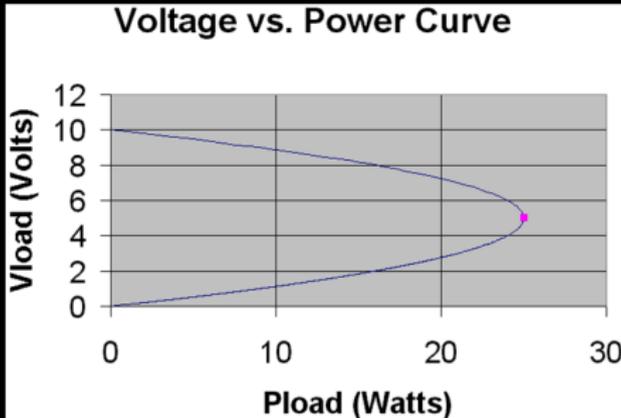
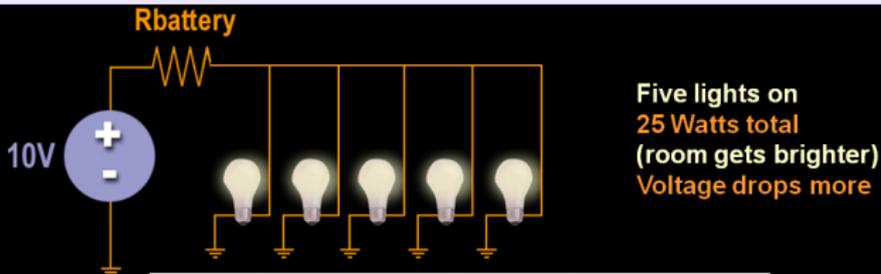
Voltage vs. Power Curve



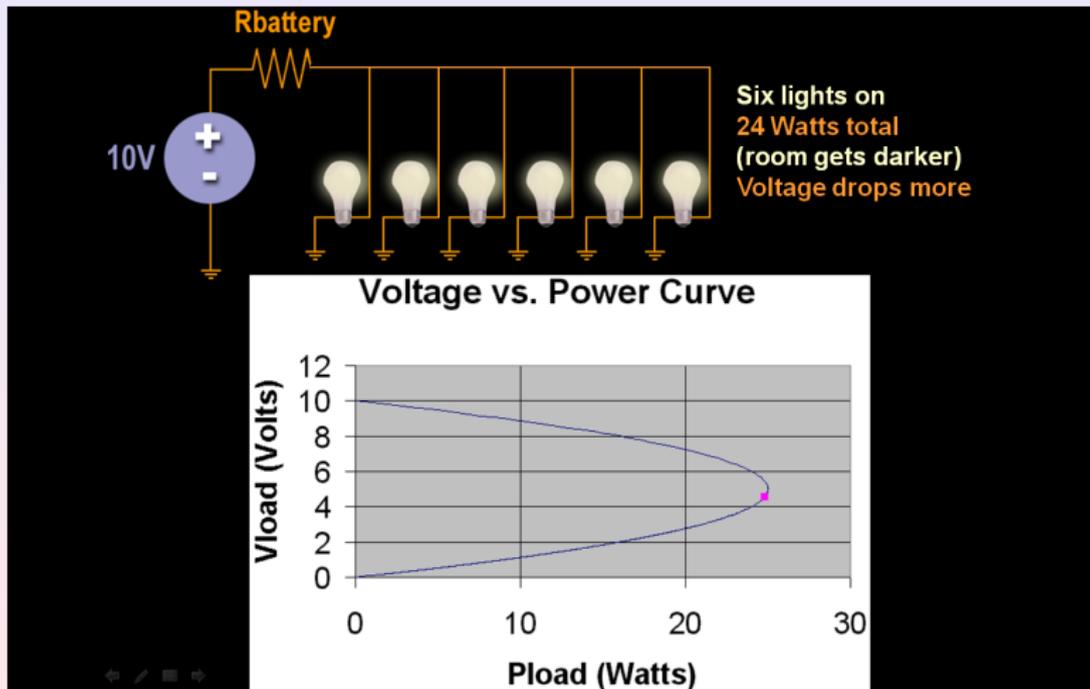
◀ Power Flows



Voltage Collapse Animation (P.W. Sauer)



Voltage Collapse Animation (P.W. Sauer)



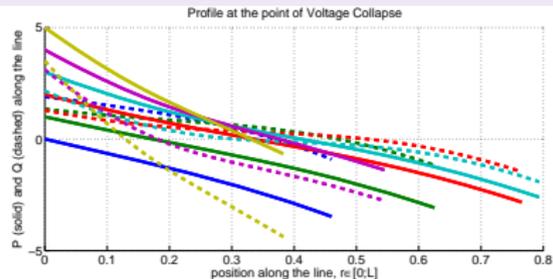
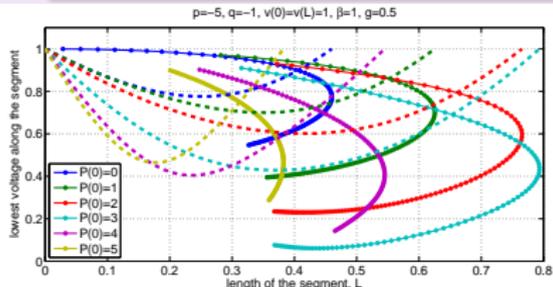
◀ Power Flows



Linear Segment of Transmission: Nose Curve

Shooting simulations: transmission segment of varying length

$v(0) = 1, \theta(0) = 0$; $P(0)$ is fixed; p, q are const; stop at $v(L) = 1$



- “Nose curve” shape (voltage collapse) is universal
- The stable solution corresponds to higher throughput (of both real and reactive)
- Position of the nose is a non monotonic function of the parameters. The line is the longest for zero throughput (most symmetric) case

DC [linearized] approximation (for AC power flows)

- (0) The amplitude of the complex potentials are all fixed to the same number (unity, after trivial re-scaling): $\forall a : \rho_a = 0$.
- (1) $\forall \{a, b\} : |\theta_a - \theta_b| \ll 1$ - phase variation between any two neighbors on the graph is small
- (2) $\forall \{a, b\} : r_{ab} \ll x_{ab}$ - resistive (real) part of the impedance is much smaller than its reactive (imaginary) part. Typical values for the r/x is in the $1/27 \div 1/2$ range.

It leads to

- Linearized relation between powers and phases (at the nodes):

$$\forall a \in \mathcal{G}_0 : p_a = \sum_{b \sim a} \frac{\theta_a - \theta_b}{x_{ab}}$$

- Losses of real power are zero in the network (in the leading order) $\sum_a p_a = 0$
- Reactive power needs to be injected (lines are inductances - only “consume” reactive power=accumulate magnetic energy per cycle)

Model of Load Shedding

Minimize Load Shedding = Linear Programming for DC

$$LP_{DC}(\mathbf{d}|\mathcal{G}; \mathbf{x}; \mathbf{u}; \mathbf{P}) = \min_{\mathbf{f}, \varphi, \mathbf{p}, \mathbf{s}} \left(\sum_{a \in \mathcal{G}_d} s_a \right)_{COND(\mathbf{f}, \varphi, \mathbf{p}, \mathbf{d}, \mathbf{s}|\mathcal{G}; \mathbf{x}; \mathbf{u}; \mathbf{P})}$$

$$COND = COND_{flow} \cup COND_{DC} \cup COND_{edge} \cup COND_{power} \cup COND_{over}$$

$$COND_{flow} = \left(\forall a : \sum_{b \sim a} f_{ab} = \begin{cases} p_a, & a \in \mathcal{G}_p \\ -d_a + s_a, & a \in \mathcal{G}_d \\ 0, & a \in \mathcal{G}_0 \setminus (\mathcal{G}_p \cup \mathcal{G}_d) \end{cases} \right)$$

$$COND_{DC} = \left(\forall \{a, b\} : \varphi_a - \varphi_b + x_{ab} f_{ab} = 0 \right), \quad COND_{edge} = \left(\forall \{a, b\} : -u_{ab} \leq f_{ab} \leq u_{ab} \right)$$

$$COND_{power} = \left(\forall a : 0 \leq p_a \leq P_a \right), \quad COND_{over} = \left(\forall a : 0 \leq s_a \leq d_a \right)$$

φ - phases; f - power flows through edges; x - inductances of edges

◀ Instantons

Instantons for Wind Generation

Setting

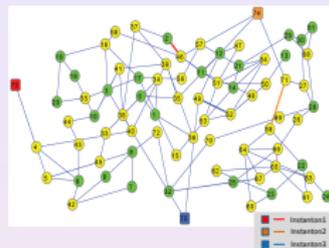
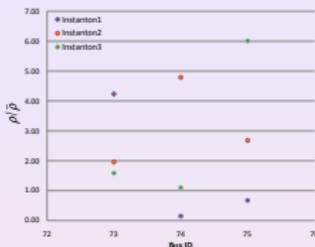
- Renewables is the source of fluctuations
- Loads are fixed (5 min scale)
- Standard generation is adjusted according to a droop control (low-parametric, linear)

Results

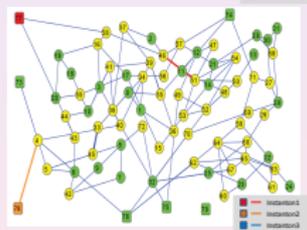
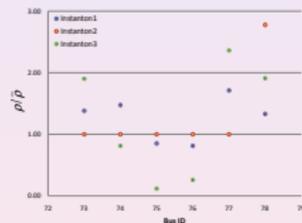
- The instanton algorithm discovers most probable UNSAT events
- The algorithm is EXACT and EFFICIENT (polynomial)
- Illustrate utility and performance on IEEE RTS-96 example extended with additions of 10%, 20% and 30% of renewable generation.

Simulations: IEEE RTS-96 + renewables

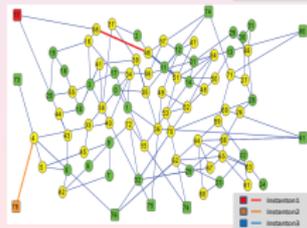
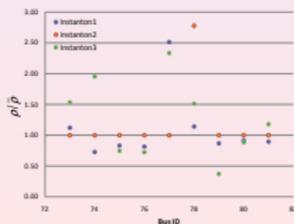
10% of penetration -
localization, long
correlations



20% of penetration - worst
damage, leading instanton
is delocalized



30% of penetration -
spreading and diversifying
decreases the damage,
instantons are localized



◀ Load Contingency